Introduction to AI

Lecture 15
Inference in First-Order Logic

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Inference in First-Order Logic

Inference in First-Order Logic is used to deduce new facts or sentences from existing sentences. Before understanding the FOL inference rule, let's understand some basic terminologies used in FOL.

Substitution:

Substitution is a fundamental operation performed on terms and formulas. It occurs in all inference systems in first-order logic. The substitution is complex in the presence of quantifiers in FOL. If we write **F[a/x]**, so it refers to substitute a constant "**a**" in place of variable "**x**".

Note: First-order logic is capable of expressing facts about some or all objects in the universe.

Equality:

First-Order logic does not only use predicate and terms for making atomic sentences but also uses another way, which is equality in FOL. For this, we can use **equality symbols**. **Example: Brother (John) = Smith.**

the object referred by the **Brother (John)** is similar to the object referred by **Smith**. The equality symbol can also be used with negation to represent that two terms are not the same objects. **Example: ¬(x=y) which is equivalent to x ≠y.**

FOL inference rules for quantifier

- As propositional logic we also have inference rules in first-order logic, so the following are some basic inference rules in FOL:
  - **Universal Generalization**
  - **Universal Instantiation**
  - **Existential Instantiation**
  - **Existential introduction**

1. Universal Generalization:

•Universal generalization is a valid inference rule that states that if premise P(c) is true for any arbitrary element c in the universe of discourse, then we can have a conclusion as ∀ x P(x) are true.

•It can be represented as: $\frac{P(c)}{\forall x\, P(x)}$

•This rule can be used if we want to show that every element has a similar property.

•In this rule, x must not appear as a free variable.

Example: Let's represent, P(c): "**A byte contains 8 bits**", so for **∀ x P(x)** "**All bytes contain 8 bits.**", it will also be true.

FOL inference rules for quantifier

2. Universal Instantiation:

- Universal instantiation also called universal elimination is a valid inference rule. It can be applied multiple times to add new sentences.
- The new KB is logically equivalent to the previous KB.
- As per UI, **we can infer any sentence obtained by substituting a ground term for the variable**.
- The UI rule states that we can infer any sentence P(c) by substituting a ground term c (a constant within domain x) from **∀ x P(x) for any object in the universe of discourse**.
- It can be represented as: $\frac{\forall x\, P(x)}{P(c)}$

Example:1.

IF "Every person likes ice cream"=> ∀x P(x) so we can infer that "John likes ice cream" => P(c)

FOL inference rules for quantifier

Example: 2.

Let's take a famous example,

"All kings who are greedy are Evil." So let our knowledge base contains this detail as in the form of FOL:

∀x king(x) ∧ greedy (x) → Evil (x),

So from this information, we can infer any of the following statements using Universal Instantiation:

King(John) ∧ Greedy (John) → Evil (John),

King(Richard) ∧ Greedy (Richard) → Evil (Richard),

King(Father(John)) ∧ Greedy (Father(John)) → Evil (Father(John))

FOL inference rules for quantifier

3. Existential Instantiation:

- Existential instantiation is also called Existential Elimination, which is a valid inference rule in first-order logic.
- It can be applied only once to replace the existential sentence.
- The new KB is not logically equivalent to the old KB, but it will be satisfiable if the old KB was satisfiable.
- What this rule says is that if P holds for some element of the universe, then we can give that element a name such as c (or x, y, a etc). When selecting symbols, one must select them one at a time and must not use a symbol that has already been selected within the same reasoning/proof.
- The restriction with this rule is that c used in the rule must be a new term for which P(c) is true.

It can be represented as: $$\frac{\exists x\, P(x)}{P(c)}$$

Example:

If you get a 95 on the final exam, then you get an O for the course. Someone, called c, gets 95 on the final exam. Therefore c gets an O. This argument uses Existential.

FOL inference rules for quantifier

4. Existential introduction

- An existential introduction is also known as an existential generalization, which is a valid inference rule in first-order logic.
- This rule states that if there is some element c in the universe of discourse that has a property P, then we can infer that there exists something in the universe that has the property P.
- It can be represented as:

$$\frac{P(c)}{\exists x P(x)}$$

- **Example: Let's say that,**

“X got good marks in English."

"Therefore, someone got good marks in English."

Generalized Modus Ponens Rule

For the inference process in FOL, we have a single inference rule which is called Generalized Modus Ponens. It is a lifted version of Modus ponens.

Generalized Modus Ponens can be summarized as " P implies Q and P is asserted to be true, therefore Q must be True."

According to Modus Ponens, for atomic sentences pi, pi', q. Where there is a substitution θ such that SUBST (θ, pi',) = SUBST(θ, pi), it can be represented as:

$$\frac{p1', p2', \ldots, pn', (p1 \land p2 \land \ldots \land pn \Rightarrow q)}{SUBST(\theta, q)}$$

Example:

We will use this rule for Kings are evil, so we will find some x such that x is king, and x is greedy so we can infer that x is evil.

Here let say, p1' is king(John) p1 is king(x)
p2' is Greedy(y) p2 is Greedy(x)
θ is {x/John, y/John} q is evil(x)
SUBST(θ, q) is evil(John).

Exercise

- From Likes(Jerry, IceCream) it seems reasonable to infer ∃x Likes(x, IceCream). Write down a general inference rule, **Existential Introduction**, that sanctions this inference. State carefully the conditions that must be satisfied by the variables and terms involved.
- Suppose a knowledge base contains just one sentence, ∃x AsHighAs(x,Everest). Which of the following are legitimate results of applying Existential Instantiation?
  - a. AsHighAs(Everest , Everest).
  - b. AsHighAs(Kilimanjaro, Everest).
  - c. AsHighAs(Kilimanjaro, Everest) ^ AsHighAs(BenNevis, Everest)
  - (after two applications).

Exercise

- Write down logical representations for the following sentences, suitable for use with
  - Generalized Modus Ponens:
  - a. Horses, cows, and pigs are mammals.
  - b. An offspring of a horse is a horse.
  - c. Bluebeard is a horse.
  - d. Bluebeard is Charlie’s parent.
  - e. Offspring and parent are inverse relations.
  - f. Every mammal has a parent.